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A METHODOLOGY FOR EVALUATING MINE ACTUATION DATA

by

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A METHODOLOGY FOR EVALUATING MINE ACTUATION DATA

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Submitted in partial fulfillment
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This thesis develops a methodology for evaluating mine actuation data. Computer models are developed to analyze actuation data obtained from the Mine Warfare Command by fitting various types of actuation curves to the data. For each actuation curve type, maximum likelihood estimates are used to determine those parameters resulting in the greatest probability of obtaining the observed data.

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THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

EXECUTIVE SUMMARY

In mine warfare, a commander's ability to make the correct decisions while conducting mine countermeasure (MCM) operations could mean the difference between life and death for those going in harm's way. Over the years, a number of computer models have been developed to assist commanders in making those difficult decisions. However, the outputs generated by these useful tactical decision aids are only as good as the tactical parameters they used for inputs. The analytical techniques used for estimating the input parameters are crucial to the success of MCM operations. Two of the more commonly required input parameters are actuation width (A) and actuation probability (B).

This thesis develops a methodology for the evaluation of mine actuation data. The analytical approach used produces maximum likelihood estimates of A and B. Separate analysis methods are also developed to test the symmetry/asymmetry of actuation data and to estimate the mean and standard deviation of a ship's navigational error.

The resulting methodology is used to analyze mine actuation data provided by the Mine Warfare Command. The data were obtained during an MCM exercise in a realistic scenario. The analysis concludes that this data set is not statistically inconsistent with the assumption that the observed data was generated from a symmetric actuation curve. The mean and standard deviation of the minesweeper's navigational error are estimated to be 19.74 yards and 159.2 yards respectively. The data was fit to three different types of generalized actuation curves with the following results:

	Rectangular Symmetrical Actuation Curve	Rectangular Asymmetrical Actuation Curve	Washburn Actuation Curve
A (Yds)	2250	2125	2250
B	.3388	.3333	.3400
C	NA	NA	100
Likelihood	2.25×10^{-13}	2.25×10^{-13}	2.25×10^{-13}

Note: C in the table above is a parameter for the Washburn Actuation Curve.

ACKNOWLEDGMENT

I would like to dedicate this work to my wife, and to express my appreciation for her loving support throughout this course of study. I couldn't have done it without you. I also thank my children Mitch and Michael for providing me the motivation to always do my best and for filling my days with joy.

I wish to thank my advisor, Professor James Eagle, for his guidance and technical advice. His academic knowledge coupled with his dedication were instrumental in the completion of this work. To my second reader, William Kroshl, I express my gratitude for his scrutiny of this thesis during the editing phase. I thank Professor Alan Washburn for suggesting the MLE technique used in this thesis. Special thanks to Mr. George Pollitt of the Mine Warfare Command for providing the data and answering numerous technical questions. Finally I wish to express my sincere gratitude to CAPT Frank Petho for his help and support during one of life's little curve balls.

I. INTRODUCTION

A. BRIEF HISTORY

> The underwater mine, it can be said, came of age with the First World War, it matured during the Second World War, and now its future as one the principal weapons of attrition and defence seems assured. [Ref. 1]

Throughout history naval mines have been effectively used to deny the enemy presence within coastal waters and to ensure friendly control of strategic waterways. The use of naval mines as a weapon is a concept which dates back to 668 A. D. In more recent times, naval mines have played a significant role in every major war involving the United States.

During World War I, the Germans successfully employed defensive and offensive mine warfare against the British. Most notable were the results obtained from the German minefields laid west of Heligoland. In the Battle of Jutland the British decided not to follow the German Fleet into the Heligoland Bight because of their concern over the German minefields. In a similar fashion, the British also benefited from the use of naval mines during the war. They were very successful in denying German submarines the use of the tactically significant Dover Straits. British minefields consisting of over 5,000 mines forced the Germans to discontinue the use of these straits and reach the Atlantic through the longer northern route, thus alleviating British merchant shipping losses.[Ref. 2]

World War II brought about significant advances in mine technology. New developments such as more sophisticated firing mechanisms, safety and delay mechanisms, and ship counters made mine warfare an even more serious threat. A total of over 300,000

mines were laid by the United States and Great Britain during the war. The total loss of enemy shipping attributed to mining projects is estimated to be close to 2,700 ships [Ref. 3]. Post-war information received from senior Japanese naval officers indicated that the mining of Japan and the numerous losses of merchant vessels due to mines were factors which greatly influenced the outcome of the war.

The delay of D-Day at Wonsan during the Korean War clearly highlights the significance of possessing a well-trained, well-equipped mine countermeasures (MCM) force capable of conducting swift and effective mine clearance operations. Wonsan served to unveil the U.S. Navy's limited capabilities in shallow-water mine clearance operations. During the minesweeping operations at Wonsan, the United States suffered the loss of four minesweepers, one fleet tug, and had five destroyers severely damaged by mine blasts. A massive minefield containing an estimated 3,000 mines paralyzed a fleet of 250 ships carrying 50,000 Marines off the coast of Wonsan for one week [Ref. 4]. After the events at Wonsan, Admiral Forrest P. Sherman, Chief of Naval Operations stated:

> When you can't go *where* you want, *when* you want to, you haven't got command of the sea. And command of the sea is a rock bottom foundation of all our war plans. We've been plenty submarine-conscious and air-conscious. Now we're going to start getting mine-conscious--beginning last week.[Ref. 5]

The vast majority of MCM operations during the Vietnam War took place in the rivers of Vietnam. The North Vietnamese relied heavily on the mining of key inland waterways to control their use for logistical purposes. The shallow water MCM operations in Vietnam were aggravated by the Vietnamese use of combined gunfire and rocket attacks

against MCM forces, making sweeping operations more difficult and dangerous than they were to begin with. One of the major mining campaigns during the war was the U.S. mining of Haipong and other North Vietnamese harbors. The results of the almost 8,000 mines laid were immediate and effectively stopped all ship traffic in or out of the harbors for ten months.

History repeated itself during the Gulf War against Iraq. In preparation for an amphibious assault, coalition forces were ordered to sweep a channel in order to provide safe passage for a battleship to a Fire Support Area off the coast of Kuwait. During the clearance operation the flagship of all MCM forces in the Gulf and its anti-air warfare coverage ship were struck by mines. The Chief of Naval Operations, Admiral Frank Kelso best summarized the events when he said:

> We recently relearned some hard lessons-- how mines can frustrate even the most powerful of naval forces. During Operation Desert Storm, Iraq's extensive minefields all but stymied a planned amphibious strike to liberate Kuwait.[Ref. 6]

B. OBJECTIVE

The main objective of this thesis is to explore the use of a fresh analytical approach for the evaluation of exercise mine actuation data. An analysis is performed of actuation data obtained during an MCM exercise in a realistic scenario. Maximum likelihood estimates (MLE) are used to determine those values of actuation width (A), and probability of actuation (B) which best represent the exercise data. A separate analysis is used to determine the standard deviation (σ) and mean (μ) of the navigational error which were needed for the MLE analysis.

C. OUTLINE

Chapter II introduces general concepts in mine warfare. Chapter III reviews preliminary analysis techniques used by the Mine Warfare Command, discusses the analysis methods used in this thesis, and presents the results of the analysis conducted. Chapter IV concludes the thesis work and comments on areas of possible future work.

II. BACKGROUND

> The effectiveness of the (submarine) mine has not decreased with the coming of the space age. So long as cargo ships cross the sea, this unspectacular weapon will remain a major factor in control of the approaches to harbors, and the shallow straits between seas.[Ref. 7]

A. MINE WARFARE

Due to its relative low cost and high effectiveness, the use of mines is a tactic which we are likely to see in future wars. Today's global political, economic, and military conditions have forced the U.S. Navy to shift its attention from blue water operations to the littorals. And it is in the world's littorals where the capability of conducting effective and thorough mine warfare becomes critical. Admiral Kelso's viewpoint : "Effective offensive and defensive mine warfare underlies--literally--the success of littoral military operations [Ref. 8].", is representative of the importance afforded to mine warfare by the Navy's senior leadership. Mine warfare serves two main objectives:

- To damage or destroy enemy shipping.
- To deny the enemy use of certain waters, or at least hinder his operations in these waters by the threat presented by a minefield.

1. Types of Mines

Naval mines are commonly grouped into two main categories:

a. Controlled mines

Controlled mines are those whose detonation is electrically controlled from

a shore site. These mines can be set to *off* to allow safe passage of friendly shipping, or they can be set to *on* to be used against enemy shipping.

b. Independent mines

Independent mines act independently once laid and do not require user intervention for detonation. They are equally lethal to all shipping regardless wether they are enemy, neutral, or friendly ships. Independent mines can be further classified by the position they occupy in the water.

(1) Moored mines. This type of mine is buoyant and floats at a predetermined depth below the surface. It is held in position by a cable attached to an anchor.

(2) Ground mines. Also known as bottom mines, these mines lay on the bottom of the ocean. They are very effective against shallow water shipping and pose a significant threat to submarines in deep water.

(3) Drifting mines. These are mines which move freely at or near the surface of the ocean. They are buoyant or neutrally buoyant, and can be attached to a float line at a set depth beneath the surface.

(4) Creeping mines. A type of drifting mine held below the surface by means of a length of wire or chain which drags along the bottom.

(5) Oscillating mines. These mines are free floating with a predetermined range of upper and lower depth below the surface. They use compressed air or gas to maintain their position in the water.

Furthermore, independent mines can also be categorized by the type of actuation mechanism which they use to detonate themselves:

(6) Contact mines. Mines which detonate only through physical contact with a ship.

(7) Magnetic mines. These mines are actuated when they detect a disturbance in the earth's magnetic field, such as the one caused by a steel hull vessel.

(8) Acoustic mines. These mines possess hydrophones which are tuned to detect mechanical noise made by shipboard machinery. They are actuated when noise within a specified frequency band is detected.

(9) Pressure mines. These mines are actuated by sensing the pressure variations in the water directly underneath a moving ship.

(10) Combination mines. These mines utilize at least two of the three basic influence mechanisms (magnetic, acoustic, or pressure). They are actuated when all its influence actuation criterion are met.

2. Minefields

There are two basic kinds of minefields:

a. Defensive

The early minefields were basically defensive in nature. These fields were designed with the main objective of keeping enemy ships out and protecting friendly shipping. This objective is still valid for present time minefield planning. Defensive minefields are mainly used in friendly harbors and strategic waterways under friendly control. The idea is to allow safe passage of friendly vessels while denying the enemy the

opportunity to position itself to conduct attacks, invasions, or disruptions of friendly operations. The mining of Wonsan during the Korean War is an excellent example of how effective defensive minefields can be against the enemy.

b. Offensive

Offensive mining is a concept which was spurred by the ability to lay mines from aircraft and submarines. In the early days of mine warfare any offensive mining had to be done from surface ships, which have a more limited ability to operate in opposed waters. These minefields are used to attack enemy shipping and to deny the enemy use of its harbors and waterways. As such, offensive minefields are laid in waters controlled by the enemy. During the Gulf War, Iraq forced American commanders to reconsider a planned amphibious assault when two U.S. warships struck mines laid by Iraq in the waters off the Kuwaiti coast.

3. Mine Countermeasures

There are two basic methods employed during the conduct of mine countermeasures:

- Use of special equipment to reduce the ship's signature.
- Physical removal or disarmament of the mine.

a. Signature reduction

Modern warships have a limited capability to actively protect themselves against various types of mines. In the case of magnetic mines, today's ships attempt to counter the threat by means of degaussing coils. Degaussing coils are intended to counteract the disturbances a ship constructed out of steel produces in the earth's magnetic field.

Against acoustic mines, engineering technological advances such as sound mounts and dampening materials have made it possible to build engines and machinery which operate at much lower noise levels than in the past. The only countermeasure in use against pressure mines is simply the reduction of ship's speed in an attempt to minimize water disturbances caused by a ship's motion through the water. Moored mines are in general easier to defeat as they can be detected by sonar, lookouts, or helicopters.

b. Physical removal or disarmament

As occurred during the Gulf War, naval vessels are sometimes ordered to operate in waters which are known or suspected to be mined. In such cases, essential routes through the water are first sanitized by conducting minesweeping and minehunting operations.

(1) Minesweeping. Minesweeping operations are tailored to the specific type of mine being swept. For influence mines, either magnetic or acoustic devices are used to simulate a ship's influence field. The acoustic devices are underwater mechanisms which produce noises similar to shipboard engines and machinery. In principle, the object is to fool the mine's acoustic sensors causing it to actuate and explode. The magnetic influence devices are essentially an electrically charged cable being towed from a ship or a helicopter. With this type of device the goal is to create an electromagnetic field strong enough to disturb the vertical component of the earth's magnetic field and thus actuate the mine. For moored mines, a paravane device is used to support a cable at its outer end while holding it out at an angle to the sweeper. As the cable is towed through the water, cutting blades attached to it cut the mooring lines of mines laying in its path. The mines then

surface and are destroyed by Explosive Ordnance Disposal personnel.

(2) Minehunting. Minehunting refers to the use of sophisticated sonar equipment to identify minefields and accurately classify the mines within them. This is a tedious and extremely difficult operation whose success is not only dependent on the sensitivity of the sonar system, but is very much influenced by the environmental and ocean floor conditions. Minehunting had its origins during World War II, where one of the preferred methods employed to locate mines was to have mine watchers guard key waterways to spot the position where mines hit the water after having been laid by aircraft. Other possible techniques for minehunting included the use of radar to fix splash positions, and the use of a hydrophone network to determine splash locations by computing time differences between sound arrivals at various hydrophones. Since then, various types of minehunting sonars have been developed to find mines and neutralize them after they have been deployed and have reach their resting location in the water. The U.S. Navy currently has a class of ships designated as coastal minehunters (MHC) which are equipped with the latest sonar and mine neutralization systems. Research and development efforts are currently ongoing for a new class of autonomous underwater vehicle to be used in a minehunting role [Ref. 9].

III. ANALYSIS METHODOLOGY AND RESULTS

A. EXERCISE DESCRIPTION

The data analyzed in this thesis were obtained during a minesweeping exercise conducted in 1984. Even though these data are quite old, they were used because sweeping procedures and mine actuation mechanisms have not significantly changed, and the data show a puzzling asymmetry which initially suggested that the minesweeper was considerably more effective on the port side than on the starboard side. This hypothesis of asymmetry was eventually shown not to be statistically supportable by the exercise data.

The first step in evaluating the mine actuation data was to gain an understanding of the physical setting of the exercise, the information being represented by the data, and the data collection procedures. Appendix A contains the raw data as it was received from the Mine Warfare Command.

Five influence exercise mines were laid in a simulated minefield measuring 1200 yards in width by 3700 yards in length. The mines were deployed via small boat by an Explosive Ordnance Disposal team using precise navigation equipment (Mini-Ranger station). The navigational equipment provided an accuracy of 2-3 meters in the placement of the mines which allowed us to assume that any errors associated with the mine locations were negligible. As shown in Figure 1, the mines were laid at equal intervals along a diagonal line at a water depth of 15-19 fathoms. After the mines were in place, a minesweeper made a total of ten north/south parallel sweeps across the width of the minefield. For each individual mine encounter with the minesweeper, post-exercise data

collection included actuation result (actuation or non-actuation), mine position relative to ship's track (left or right), lateral range between the ship and the mine (i.e., the closest point of approach), and time of actuation. The actuation result and time of actuation information were obtained from data recording systems housed within each mine. The relative position and lateral range information were reconstructed after the exercise using the ship's navigational charts.

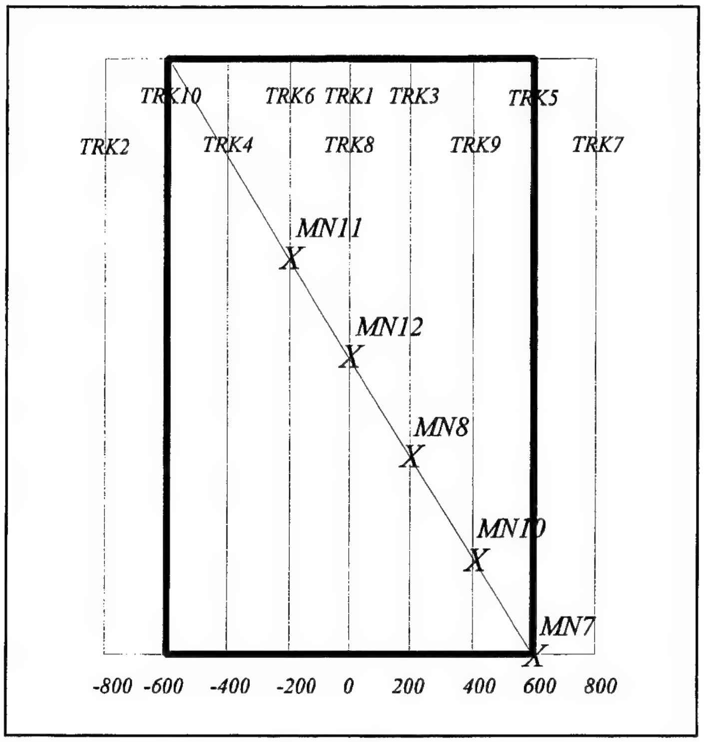

Figure 1. Simulated Minefield.

B. ACTUATION CURVES

1. Idealized Actuation Curves

An actuation curve is a plot of the probability that a ship passing a mine at some lateral range *(d)* will cause the mine to actuate. The lateral range *d* is the distance between the mine and the ship at the closest point of approach. The ship's track is assumed to be infinite in both directions and to contain no course changes. It is important to understand that an actuation curve is not a probability density function (PDF) nor a cumulative distribution function (CDF). Actuation curves simply represent the cumulative probability of actuation sometime along the ship's track and under a particular set of environmental conditions.

For this analysis, the exercise data was fit to three types of idealized actuation curves: a symmetrical rectangular actuation curve, an asymmetrical rectangular actuation curve, and Washburn's actuation curve [Ref. 10]. Figure 2 shows examples of each type of actuation curve used in the analysis.

C. PRELIMINARY ANALYSIS

During the 1950's R. K. Reber conducted valuable research work on mine warfare issues. Much of his work and conclusions have become the basis for current analysis methodologies in the mine warfare field. One such methodology is the NATO Standard Naval Agreement (STANAG) 1142. This NATO Agreement delineates a procedure for estimating actuation width (*A*) and actuation probability (*B*) from mine actuation data.

The Mine Warfare Command uses this methodology to conduct preliminary analysis of field data. The analysis conducted in this thesis is aimed at exploring alternative data reduction procedures.

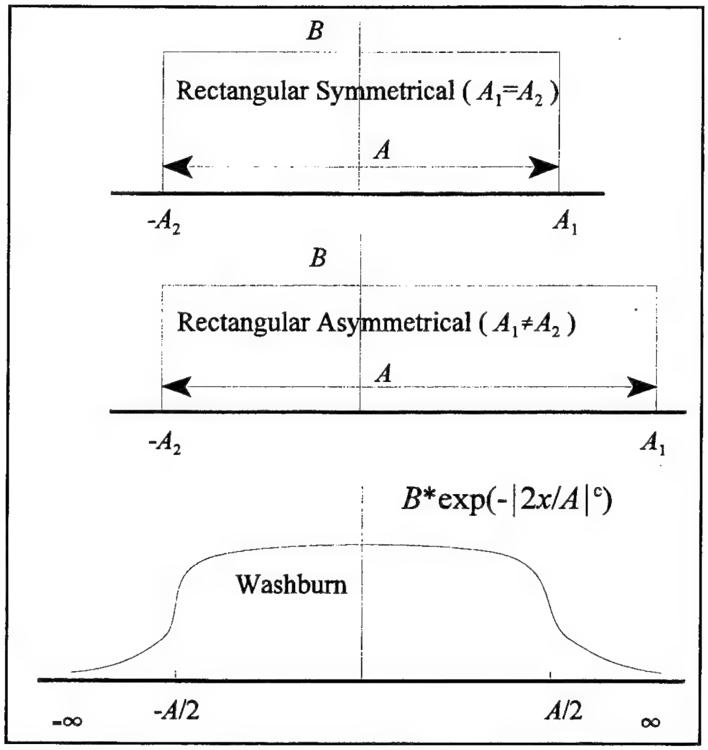

Figure 2. Types of Actuation Curves.

1. Empirical Actuation Curve

The preliminary analysis performed on the exercise data (at the Mine Warfare Command in 1985) consisted of estimating A and B using empirical techniques from STANAG 1142. The underlying idea is to construct a histogram using field data to provide an approximation of the actual actuation curve. The histogram is generated by partitioning

the lateral range of the data into intervals of equal length, and determining the empirical probability of actuation for each interval. The probability of actuation for any given lateral range interval is calculated by dividing the number of mine actuations in a particular lateral range interval (x) by the total number of mine encounters in that particular interval (n). In this context, a mine encounter is a single pass between a ship and a mine. Mine encounters are completely characterized by the lateral range between the ship's track and the mine, and the actuation outcome. Negative lateral ranges result when the mine passes on the port side of the ship. Each time a mine encounter takes place, the outcome is either an actuation or a non-actuation.

Figures 3a and 3b show two possible empirical actuation curves based on the exercise data used in this thesis. In Figure 3a, a left-right symmetry is assumed by considering only the absolute value of the lateral range. Figure 3b shows the empirical actuation curve where a distinction is made between mines passing on the left or the right of the minesweeper. In the asymmetric curve it can be seen that the majority of the mine actuations occurred on the left side of the minesweeper. Upon careful review of the exercise procedures, it was discovered that the asymmetry in this curve was caused in large part by the design of the exercise. Assuming perfect navigation, the minesweeper would have seven opportunities to pass mines off its starboard side, 37 opportunities on the port side, and six opportunities with a lateral range of zero. From the reconstructed tracks, 11 mines actually passed off the starboard side, 37 off the port side, and 2 down centerline. This helps to explain why the majority of mine encounters and mine actuations were reported as having occurred on the left side of the minesweeper.

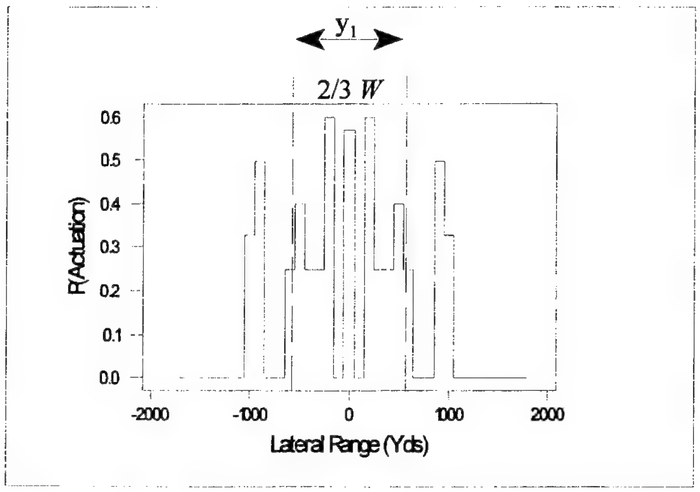

Figure 3a. Empirical Actuation Curve of Exercise Data Assuming Symmetry.

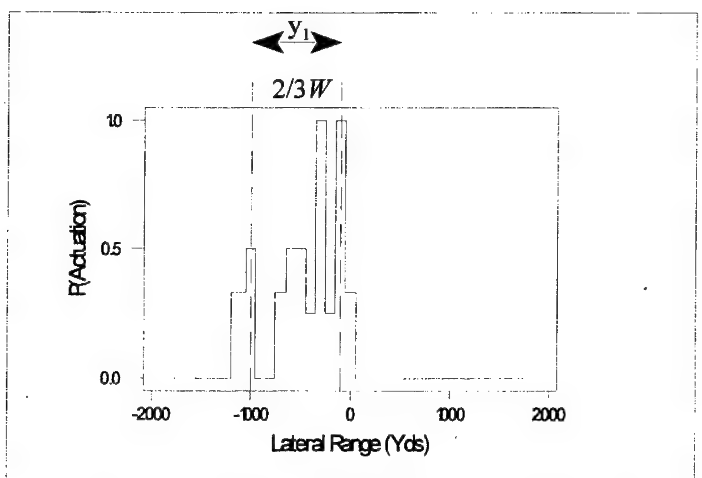

Figure 3b. Empirical Actuation Curve of Exercise Data Assuming Non-Symmetry

The undersampling that occurred in the starboard range bins causes some complications in data analysis. The actuation data in the asymmetric curve in Figure 3 suggest that the minesweeper had very little effectiveness on its starboard side, but this might be explainable by the small sample sizes obtained on the starboard side. Intuition suggests that the empirical actuation curve should be approximately symmetric. In the remainder of this subsection, we develop confidence regions for the actuation proportions observed in each range bin. The intent of this development is to show that the assumption of a symmetrical actuation curve is not statistically inconsistent with the observed data.

Letting p be the underlying actuation probability for some range bin, $\hat{p}$ (a point estimate for p) can be determined by dividing the number of actuations observed in that range bin (x) by the total number of observations in that bin (n). We intend to calculate an approximate 100(1-α)% confidence region about $\hat{p}$. The confidence region is defined by $\hat{p}_L$ and $\hat{p}_U$, where:

- $\hat{p}_L$ satisfies $P(X \geq x) = \alpha/2$, where X is a binomial random variable with parameters $\hat{p}_L$ and n.

- $\hat{p}_U$ satisfies $P(X \leq x) = \alpha/2$, where X is a binomial random variable with parameters $\hat{p}_U$ and n.

Expressing the binomial distribution explicitly,

- $\hat{p}_L$ satisfies $\sum_{i=x}^{n} \binom{n}{i} \hat{p}_L^{\,i}(1-\hat{p}_L)^{n-i} = \alpha/2$

- $\hat{p}_U$ satisfies $\sum_{i=0}^{x} \binom{n}{i} \hat{p}_U^{\,i}(1-\hat{p}_U)^{n-i} = \alpha/2$

If the true binomial actuation probability in a range bin is $\hat{p}_L$, then the probability of receiving the number of actuations actually observed or more is $\alpha/2$. That is, $\hat{p}_L$ is taken to be the smallest actuation probability which is reasonably consistent with the observed data. Similarly, if the true binomial actuation probability in a range bin is $\hat{p}_U$, then the probability of receiving the number of actuations observed or fewer is $\alpha/2$. In this case $\hat{p}_U$ is interpreted as the largest p reasonably consistent with the observed data.

For $\alpha = .1$ and .2, Figures 4a and 4b show the confidence regions for each range bin of the asymmetrical actuation curve previously presented, with an overlay of the symmetric actuation curve of Figure 3a. It can be seen that the symmetrical actuation curve is well within the boundaries of the confidence regions for these α values. Based on these confidence regions and in spite of the asymmetry of Figure 3b, we conclude that it is not unreasonable to assume that the actuation curve which generated this data is symmetric.

2. STANAG 1142 Estimate of Actuation Width and Actuation Probability

Once an empirical actuation curve has been constructed, the first step delineated in STANAG 1142 is to determine the aggregate actuation width (W) by calculating the area underneath the empirical actuation curve. The next step in the procedure is to find that distance y_1 which contains the central two thirds of the total area under the actuation curve (refer to Figure 3). The actuation probability (B) is then assumed to be the average value of the actuation curve over the central two thirds region, and the actuation width (A) is selected so that $A * B = W$. When applied to the actuation data analyzed here, STANAG 1142

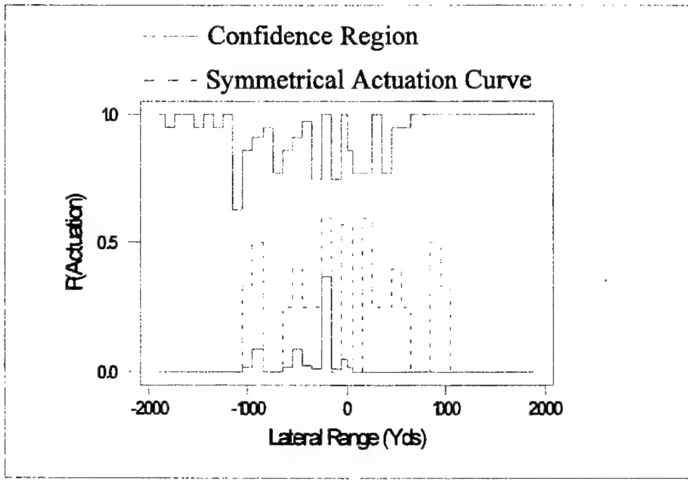

Figure 4a. Symmetrical Actuation Curve Compared with Confidence Region Derived from an Asymmetrical Actuation Curve (α=.1).

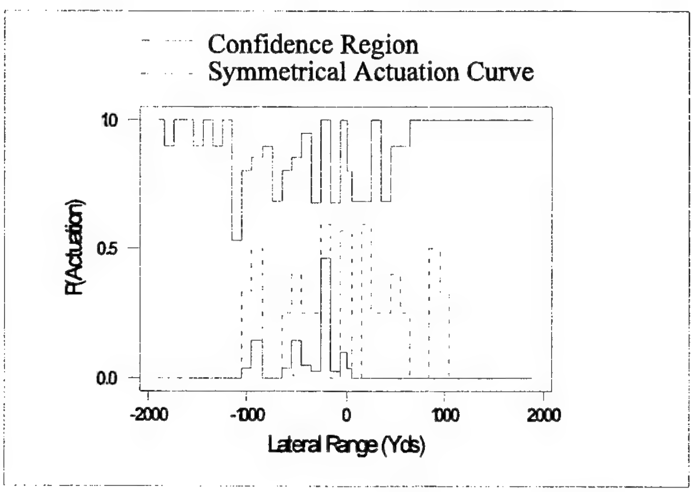

Figure 4b. Symmetrical Actuation Curve Compared with Confidence Region Derived from an Asymmetrical Actuation Curve (α= .2).

produced estimates for A and B of 1836 yards and .3432 respectively for the symmetric actuation curve of Figure 3a, and 1276.5 yards and .3909 for the asymmetric curve of Figure 3b.

D. MAXIMUM LIKELIHOOD APPROACH

For each actuation curve type, we used maximum likelihood estimates to find those parameters resulting in the greatest probability of receiving the actuation results actually observed during the exercise. But before proceeding it is necessary to estimate the minesweeper's navigational error.

1. Standard Deviation and Mean of the Ship's Navigational Error as Determined by Exercise Data

Two of the parameters needed for the likelihood analysis are an estimate of the standard deviation (σ) and mean (μ) of the ship's navigational error. For computational convenience, these two parameters were estimated by conducting a separate analysis. An examination of the exercise's physical setting suggested a simple approach for estimating these parameters. Here, *reported range* is defined as the lateral distance between the ship and a mine as determined through post-exercise reconstruction. Similarly, *predicted range* is that same distance as calculated by assuming the ship followed its intended track exactly. Figure 5 is a graphical representation of reported ranges versus the predicted ranges. A histogram of the difference between reported range and predicted range was generated, and a normal curve was fitted to it, shown in Figure 6. The resulting normal curve yielded estimates for the standard deviation and the mean of 159.2 yards and 19.74 yards respectively, with a Chi-Square value of 3.69276 which indicates a relatively good fit.

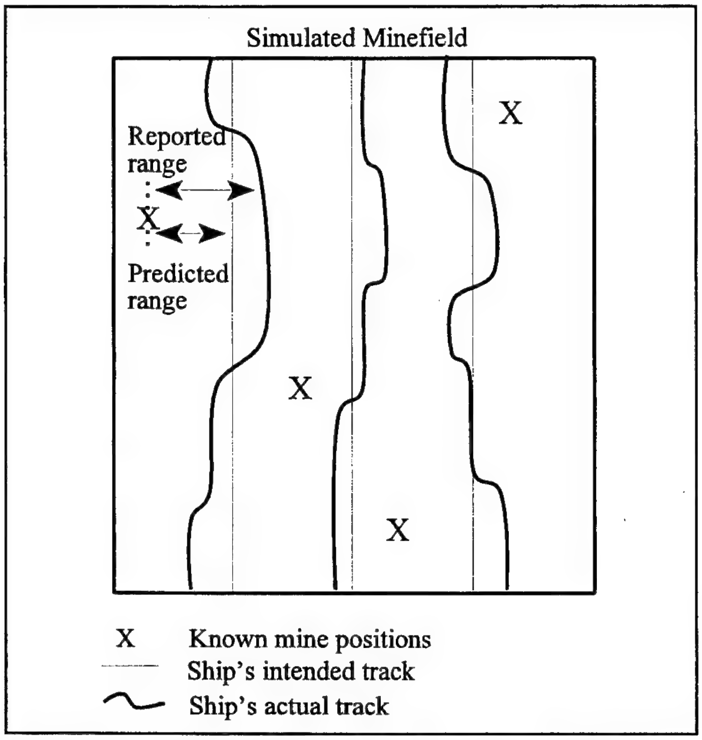

Figure 5. Reported Range versus Predicted Range.

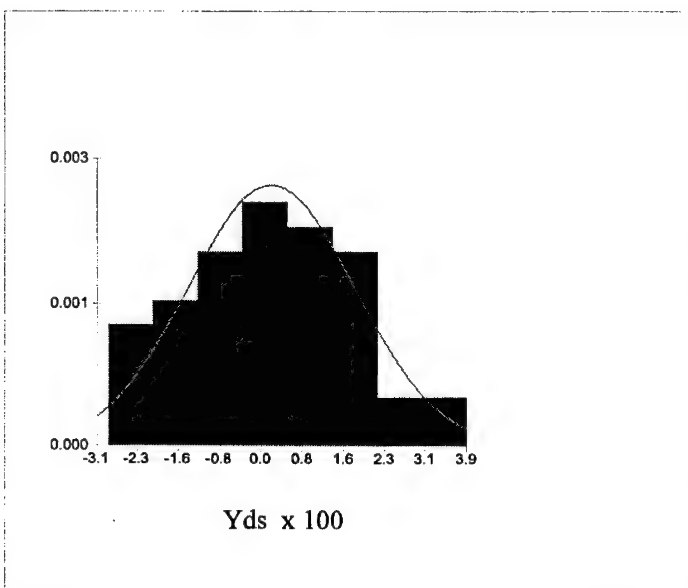

Figure 6. Normal Density Fit to Histogram of Ship's Deviations from its Intended Track.

2. Maximum Likelihood Analysis Background

An important part of the likelihood analysis was to compute the probability of an actuation given a predicted lateral range d. The eventual likelihood function was the product of these probabilities, one for each ship-mine encounter. The initial assumptions made were: normal navigational errors with standard deviation σ and mean μ, a symmetrical rectangular actuation curve with parameters A and B, and a predicted lateral range of d. Letting,

$$\mathrm{P}(\text{mine actuates} \mid \text{lateral range} \le A/2, \text{predicted lateral range} = d) = B \text{ and}$$

$$\mathrm{P}(\text{lateral range} \le A/2 \mid \text{predicted lateral range} = d) = \Phi\left(\frac{d+A/2-\mu}{\sigma}\right) - \Phi\left(\frac{d-A/2-\mu}{\sigma}\right),$$

we have

$$\mathrm{P}(\text{mine actuates} \mid \text{predicted lateral range} = d) =$$

$$B * \left[\Phi\left(\frac{d+A/2-\mu}{\sigma}\right) - \Phi\left(\frac{d-A/2-\mu}{\sigma}\right)\right]. \qquad \textbf{[1]}$$

Figure 7 is a visual representation of Equation 1. In this figure, the Gaussian curve represents the distribution of the ship's lateral range. The rectangular curve is a generic actuation curve for a mine. The probability of actuation is B times the area under the density curve within $A/2$ of the mine position.

3. Maximum Likelihood Estimate for Parameter *B*

The analytical form of Equation 1 allows us to quickly check if the maximum likelihood estimate for B is equal to one. The likelihood function is:

$$\mathrm{L}(B) = \begin{cases} B^{I} * \prod_{i=1}^{I} y_i\,, & J=0 \\ \left[B^{I} * \prod_{i=1}^{I} y_i\right] * \prod_{j=1}^{J} (1 - B * y_j), & J \ge 1 \end{cases} \qquad \textbf{[2]}$$

where I is the number of positive actuations, J is the number of negative actuations, and y_k is the probability that the ship comes within $A/2$ of the mine during mine encounter k. Equation 2 is the probability of receiving the actuation results actually obtained from the exercise, assuming probabilistic independence for each mine encounter. In order to determine the maximizing B, the log of the likelihood function was differentiated with respect to B and set equal to zero.

$$\frac{d\log[L(B)]}{dB} = \frac{I}{B} - \sum_{j=1}^{J}\left(\frac{y_j}{1-B*y_j}\right), \quad [3]$$

which gives the following when evaluated at B=1:

$$\left.\frac{d\log[L(B)]}{dB}\right|_{B=1} = I - \sum_{j=1}^{J}\left(\frac{y_j}{1-y_j}\right). \quad [4]$$

Due to the concavity of log $[L(B)]$ for $0\leq B \leq 1$, Equation 4 implies that if the number of actuations $I \geq \sum_{j=1}^{J}\left(\frac{y_j}{1-y_j}\right)$, then the slope of log $[L(B)]$ is positive at B=1 and the maximizing B (in the interval $0\leq B \leq 1$) must be one. If $I < \sum_{j=1}^{J}\left(\frac{y_j}{1-y_j}\right)$, then the maximizing B is found by numerically solving for the root of Equation 3 in the interval $0\leq B \leq 1$. The importance of this procedure is that it allows us to solve for the best B once the other model parameters have been specified. This reduces the amount of searching we have to do to find the model parameters maximizing the likelihood function.

4. Results of the Analysis

Using the theoretical background previously discussed, three MATLAB models were developed. The models were used to evaluate the exercise data and to compute those

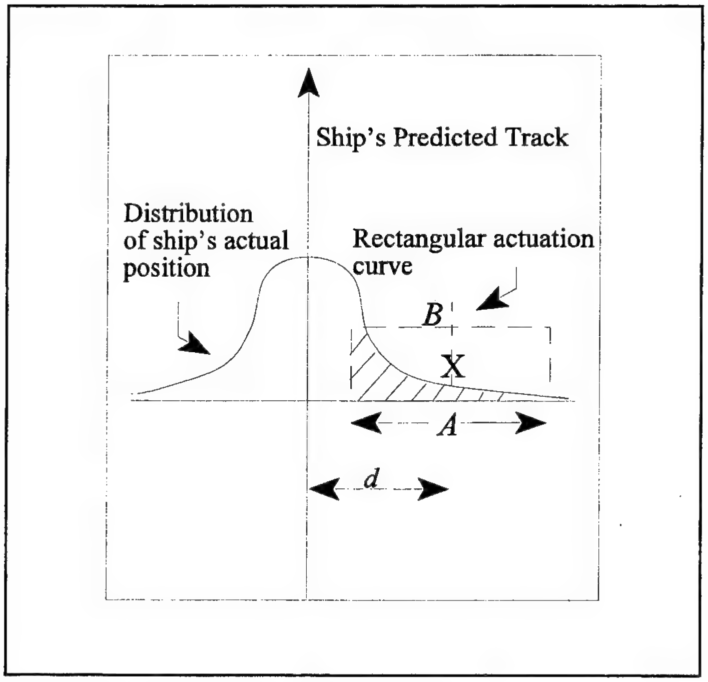

Figure 7. Probability of Actuation.

parameters corresponding to the greatest likelihood of receiving the actuation results actually obtained, had the data come from the particular actuation curve being modeled. All three models were very similar, with the major difference being in the expressions used to compute the conditional probability of a mine actuation (see Equation 1).

a. *Symmetrical Rectangular Curve*

The first and simplest case considered was a symmetrical rectangular actuation curve. In this model, Equation 1 remained unchanged from the form presented during the discussion of the theory. For this case the parameters of interest were actuation width (*A*), and actuation probability (*B*). The MATLAB code for this model is included as Appendix B, its run time was 75 seconds on a 486/33MHz PC.

b. *Asymmetrical Rectangular Curve*

The second model examined the fit of the data to an asymmetrical rectangular actuation range curve. Here Equation 1 became:

P(actuation | predicted lateral range= *d*) =

$$B * \left[\Phi\left(\frac{d+A_1-\mu}{\sigma}\right) - \Phi\left(\frac{d-A_2-\mu}{\sigma}\right)\right]. \qquad [5]$$

In this model the parameters of interest were actuation width (as defined by A_1 and A_2 in Figure 2) and actuation probability *(B)*. The MATLAB code for this model is included as Appendix C. The run time for this model was 11 minutes .

c. *Washburn's Curve*

The final case examined the use of a more complicated, yet more realistic, type of actuation curve to analyze the exercise data. Using Washburn's actuation curve the conditional probability of a mine actuation was computed as:

P(actuation | predicted lateral range = *d*) =

$$\int_{-\infty}^{\infty}\left[B * \exp\left(\left|-\frac{2x}{A}\right|^{c}\right)\right] * \left[\frac{1}{\sigma * \sqrt{2\pi}} * \exp-\left(\frac{(d-x-\mu)^2}{2\sigma^2}\right)\right] dx, \qquad [6]$$

where x is the lateral range; and A, B, and c are parameters. The MATLAB code for this model is included as Appendix D. The run time for this model was 39 minutes.

As previously discussed, the exercise data is assumed to have been generated from the symmetrical actuation data shown in Figure 3a. However, for the purpose of illustrating the use of the methodology for asymmetric data, the three models were also run under the

assumption that the data had been generated from the asymmetrical actuation data of Figure 3b. The results for both data sets are summarized below in Tables 1 and 2:

	Rectangular Symmetrical Actuation Curve	Rectangular Asymmetrical Actuation Curve	Washburn Symmetrical Actuation Curve
A (Yds)	2250	2250 (A_1= 1125, A_2= -1125)	2250
B	.3388	.3333	.3400
C	NA	NA	100 (maximum examined)
Likelihood	2.25×10^{-13}	2.25×10^{-13}	2.25×10^{-13}

Table 1. Summary of Results Using Symmetric Data.

Table 1 shows the symmetric data results. As expected, the rectangular asymmetric and the rectangular symmetric fits were essentially identical. What was more surprising was that the best Washburn fit was also rectangular. This is shown by the large value of the best fit C. Experimentation with the data revealed that more actuations with small lateral ranges increases the central tendency of the data and depresses the best fit C.

	Rectangular Symmetrical Actuation Curve	Rectangular Asymmetrical Actuation Curve	Washburn Symmetrical Actuation Curve
A (Yds)	2325	1175 (A_1= 100, A_2= -1075)	2350
B	.3390	.4286	.3376
C	NA	NA	100
Likelihood	2.24×10^{-13}	5.38×10^{-12}	2.24×10^{-13}

Table 2. Summary of Results Using Asymmetric Data.

When the three idealized actuation curves were fit to the asymmetric data, the rectangular asymmetrical actuation curve provided the best fit, as was expected. Once again, the best fit Washburn curve closely approximated the rectangular symmetrical fit.

IV. CONCLUSIONS AND RECOMMENDATIONS

This thesis develops a methodology for the evaluation of mine actuation data. It makes use of maximum likelihood estimates (MLE) theory to determine the desired parameters, for various mine actuation curves, corresponding to the largest *likelihood* of obtaining the observed data. These parameters are important in mine warfare because they are inputs to larger models used to plan and evaluate mine clearance operations. The main advantages of this methodology are its applicability to both symmetric and asymmetric data, and its conceptual simplicity. Its main drawback is the large computational effort required to maximize the log likelihood function. Separate analysis methods are also presented for testing the symmetry/asymmetry of exercise data, and for estimating the mean and standard deviation of a ship's navigational error.

The methodology presented here has a direct application to mine warfare and could be of considerable use to minefield planners and analysts. As such, further research should be conducted to expand it and exploit its potential benefits. Possible areas of future work include:

- Applying the methodology to larger data sets (both symmetric and asymmetric).
- Incorporating the methodology into existing tactical decision-aids.
- Developing a graphical user interface to facilitate the use of this methodology in the Fleet.
- Exploring the use of logistic regression to fit the empirical actuation data. [Ref. 11]

APPENDIX A. EXERCISE MINE ACTUATION DATA

Track Number	Mine 7	Mine 10	Mine 8	Mine 12	Mine 11
1 (North)	OR690	OR410	OR410	OL190	XL205
2 (South)	OL1540	OL1160	OL1170	OL675	XL680
3 (North)	OR550	OR275	OR235	XL310	OL350
4 (South)	OL1100	XL950	XL990	OL335	XL297
5 (North)	OR180	XL100	OL180	OL800	OL1850
6 (South)	OL900	OL640	OL700	OL160	XC00
7 (North)	OR90	XL210	OL340	OL900	OL1030
8 (South)	OL780	OL540	OL560	XR25	OR75
9 (North)	OR150	XL80	OC00	XL540	XL510
10 (South)	OL1340	OL1090	XL1060	OL495	XL440

O- Non-actuation
X- Actuation
R- Right
L- Left
C- Centerline
Numbers- Lateral distance in yards

Note: In the table above, the result of the mine encounter during track 1 for mine 7 was: no actuation (O), the encounter occurred on the right in relation to the ship's track (R), and the lateral distance between the mine and the ship was 690 yards (690).

APPENDIX B. MATLAB CODE FOR SYMMETRICAL RECTANGULAR CURVE USING SYMMETRIC DATA

```
clear
clc

sigma= 159.2;
mu= 19.74;
d= [690,410,410,190,205,1540,1160,1170,675,680,550,275,...
    235,310,350,1100,950,990,335,297,180,100,180,800,...
    1850,900,640,700,160,0,90,210,340,900,1030,780,...
    540,560,25,75,150,80,0,540,510,1340,1090,1060,475,...
    440];

det_out= [0,0,0,0,1,0,0,0,0,1,0,0,0,1,0,0,1,1,0,1,0,1,0,...
          0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,1,0,0,1,0,1,1,0,...
          0,1,0,1];

d_neg= [690,410,410,190,1540,1160,1170,675,550,275,235,...
        350,1100,335,180,180,800,1850,900,640,700,160,...
        90,340,900,1030,780,540,560,75,150,0,1340,1090,...
        475];

global y_neg;
Pval= 0;
min_Pval= 1000;
A= 0;
B= 0;
n_act= 15;

for i= 1:200;
   a= 0+ (i-1)*25;
   y_neg= snormcum((d_neg+(a/2)-mu)/sigma)-...
          snormcum((d_neg-(a/2)-mu)/sigma);
   n= sum(y_neg./(1-y_neg));
   if n_act > n
      b(i)= 1;
   else
      b(i)= fzero('f',.5);
   end
   Prob=b(i)*[snormcum((d+(a/2)-mu)/sigma)-...
              snormcum((d-(a/2)-mu)/sigma)];
   Pd= (det_out.* Prob) + [(1-det_out).*(1-Prob)];
   Pval(i)= sum(abs(log(Pd)));
   if min_Pval > Pval(i),
      min_Pval = Pval(i);
      A= a;
      B= b(i);
   end
end
```

```
fprintf('The likelihood is= %g.\n\n', exp(-min_Pval))
fprintf('A= %g.\n\n', A)
fprintf('B= %g.\n\n', B)
```

APPENDIX C. MATLAB CODE FOR ASYMMETRICAL RECTANGULAR CURVE USING ASYMMETRIC DATA

```
clear
clc

sigma= 159.2;
mu= 19.74;
d= [690,410,410,-190,-205,-1540,-1160,-1170,-675,-680,...
     550,275,235,-310,-350,-1100,-950,-990,-335,-297,...
     180,-100,-180,-800,-1850,-900,-640,-700,-160,0,90,...
    -210,-340,-900,-1030,-780,-540,-560,25,75,150,-80,0,...
    -540,-510,-1340,-1090,-1060,-475,-440];

det_out= [0,0,0,0,1,0,0,0,0,1,0,0,0,1,0,0,1,1,0,1,0,1,0,...
           0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,1,0,0,1,0,1,1,0,...
           0,1,0,1];

d_neg= [690,410,410,-190,-1540,-1160,-1170,-675,550,275,...
        235,-350,-1100,-335,180,-180,-800,-1850,-900,...
       -640,-700,-160,90,-340,-900,-1030,-780,-540,-560,...
        75,150,0,-1340,-1090,-475];

global y_neg;
Pval= 0;
min_Pval= 1000;
A= 0;
B= 0;
n_act= 15;

for i= 1:44;
   a1= 0+ (i-1)*25;
   for j= 1:43;
      a2= 1050- (j-1)*25;
      y_neg= snormcum((d_neg+a1)-mu/sigma)-...
             snormcum((d_neg-a2)-mu/sigma);
      n= sum(y_neg./(1-y_neg));
      if n_act > n
         b(i,j)= 1;
      else
         b(i,j)= fzero('f',.5);
      end
      Prob=b(i,j)*[snormcum((d+a1)/sigma)-...
                   snormcum((d-a2)/sigma)];
      Pd= (det_out.* Prob) + [(1-det_out).*(1-Prob)];
      Pval(i,j)= sum(abs(log(Pd)));
      if min_Pval > Pval(i,j),
         min_Pval = Pval(i,j);
         A1= a1;
         A2= a2;
```

```
        B= b(i,j);
      end
   end
end

fprintf('The likelihood is= %g.\n\n', exp(-min_Pval))
fprintf('A1= %g.\n\n', A1)
fprintf('A2= %g.\n\n', A2)
fprintf('B= %g.\n\n', B)
```

APPENDIX D. MATLAB CODE FOR WASHBURN SYMMETRICAL CURVE USING SYMMETRIC DATA

```
clear
clc

sigma= 159.2;
mu= 19.74;

d= [690,410,410,190,205,1540,1160,1170,675,680,550,275,...
    235,310,350,1100,950,990,335,297,180,100,180,800,...
    1850,900,640,700,160,0,90,210,340,900,1030,780,...
    540,560,25,75,150,80,0,540,510,1340,1090,1060,475,...
    440];

det_out= [0,0,0,0,1,0,0,0,0,1,0,0,0,1,0,0,1,1,0,1,0,1,0,...
          0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,1,0,0,1,0,1,1,0,...
          0,1,0,1];

d_neg= [690,410,410,190,1540,1160,1170,675,550,275,235,...
        350,1100,335,180,180,800,1850,900,640,700,160,...
        90,340,900,1030,780,540,560,75,150,0,1340,1090,...
        475];


x_neg= -1791.43:108.57:1900;
x= -1900:10:1900;

global y_neg;
Pval= 0;
min_Pval= 1000;
A= 0;
B= 0;
C= 0;
n_act= 15;

for i= 1:30;
   fprintf('i= %g.\n\n', i)
   a= 500+ (i-1)*25;
   c= 0;
   for j= 1:10;
     c= c+ 10;
     for h=1:35
       s= d_neg(h)-x_neg;
       a_neg= exp(-abs((x_neg/a)).^c);
       b_neg= exp(-.5*((s-mu)/sigma).^2);
       z_neg= a_neg.*b_neg;
       y_neg(h)= 1/(sigma*sqrt(2*pi))*simrule(z_neg,65.71);
     end
     n= sum(y_neg./(1-y_neg));
     if n_act > n
```

```
            b= 1;
         else
            b= fzero('f',.5);
         end
         for k= 1:50
            r= d(k)-x;
            z= exp(-abs((x/a)).^c).*exp(-.5*((r-mu)/sigma).^2);
            Prob(k)= (b/(sigma*sqrt(2*pi)))*simrule(z,10);
         end
         Pd= (det_out.* Prob) + [(1-det_out).*(1-Prob)];
         Pval= sum(abs(log(Pd)));
         if min_Pval > Pval
            min_Pval = Pval;
            A= 2*a;
            C= c;
            B= b;
         end
      end
   end

fprintf('The likelihood is= %g.\n\n', exp(-min_Pval))
fprintf('A= %g.\n\n', A)
fprintf('C= %g.\n\n', C)
fprintf('B= %g.\n\n', B)
```

LIST OF REFERENCES

1. Griffiths, M., *The Hidden Menace*, p.147, Conway Maritime Press, 1981.

2. Hartman, G. K., *Weapons That Wait,* p.43, Naval Institute Press, 1991.

3. Duncan, R. C., *America's Use of Sea Mines*, p.159, United States Naval Ordnance Laboratory, January 1962.

4. Melia, T. M., *Damn the Torpedoes: A Short History of the U.S. Naval Mine Countermeasures, 1777-1991*, p.78, Naval Historic Center, 1991.

5. Ibid, p.79.

6. Kelso, F.B., Chief of Naval Operations, "Building Blocks of Naval Power", *Naval Institute Proceedings*, p.40, November 1992.

7. Duncan, R. C., *America's Use of Sea Mines*, p.V, United States Naval Ordnance Laboratory, January 1962.

8. Kelso, F.B., Chief of Naval Operations, "Building Blocks of Naval Power", *Naval Institute Proceedings*, p.39, November 1992.

9. Honegger, B., "NPS Tests Mine-hunting Robot", *Campus News*, v. 3, p. 3, 15 February 1996.

10. Washburn, A. R., *Search and Detection*, p.4-3, Military Applications Section, Operations Research Society of America, October 1989.

11. Collett, D., *Modelling Binary Data*, p.56, Chapman & Hall, 1991.

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